



Team Round

30 minutes | 15 problems

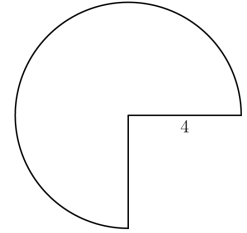
1. Alice finds three cards of the spade () suite and two cards of the club () suite lying on the path in front of her. What is the probability she arranges the cards in the order shown below?



2. When the townspeople refused to pay the Pied Piper after he got rid of the town's rat infestation, the Pied Piper decides to get rid of the child infestation. He starts playing a little tune on his pipe, and all 150 of the town's children are enchanted and follow him out of town. 32 children wore a red shirt, blue pants, and green shoes, 46 wore a red shirt and blue pants, 45 wore red shirts and green shoes, and 43 wore blue pants and green shoes. If 92 children wore blue pants and 82 children wore green shoes, how many children wore ONLY red shirts?
3. The ratio of the giant's size to Jack's is 143: 16. The ratio of Jack to the family cow he sells off is 2: 5. The ratio of the cow to the magic beans Jack gets in return for the cow is 200: 11. The ratio of the giant to the magic beans, when simplified to the highest degree, can be expressed in the form $m: n$, where m and n are both integers. What is $m + n$?
4. Write the recurring decimal $0.256256256256\dots$ as a fraction in simplest form.
5. Alice is playing a game of cards with the Mad Hatter when he suddenly asks: "How many positive integer factors are there in $13!$, the number of cards in a Hogwarts deck?" How should Alice answer?
6. When Dorothy, Scarecrow, the Tin Woodman, and the Cowardly Lion finally reach the Emerald City, they need to open the lock on the city wall's gates to pass. An inscription reads: "Convert 328 from base 12 to base 9 to pass these emerald gates." What should Dorothy say is the converted number?
7. You were wandering with Hansel and Gretel through the woods when the three of you came upon a candy house and were trapped by the Gingerbread Hag to be eaten. While the Gingerbread Hag was out, you notice that the lock on your cage can be opened if you find the value of one expression: $\sqrt{9 - 6\sqrt{2}} + \sqrt{9 + 6\sqrt{2}}$. The value of this expression can also be expressed in the simplest form $m\sqrt{n}$. What is this value?

8. When riding on his magic carpet, Aladdin notices that he's traveled 15951 miles (a palindrome, as it reads the same from left to right). Five hours later, when traveling at a steady rate, Aladdin notices that they have traveled an amount of miles that was the next highest palindrome. How far had they traveled at hour three?

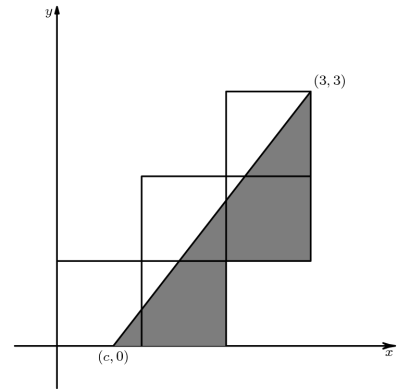
9. Peter Pan needs your help to make him a new hat. For some reason, his head has shrunk, so he cuts out a fourth of the radius 4 circle he used to make his hat. Next, he makes a new right circular cone hat by sewing the edges together. The volume of the new hat in cubic inches can be expressed as $m\pi\sqrt{n}$. What is $m + n$?



10. Goldilocks stumbled across a cabin, and, seeing the door was ajar, decided to enter. Upon reaching the dining room she notices two circular bowls of porridge (with centers A and B) that are externally tangent. Circular bowls A and B have radii of lengths 5 and 3, respectively. A line externally tangent to both of these bowls intersects ray AB at point C. What is the length of BC?

11. After Jack traded in his family cow for 7 magic beans, he noticed that these beans had numbers on them: 1, 2, 5, 7, 8, and 13. The seventh bean had the variable X marked on it. The beans told Jack that the average of the beans is the value of one of the numbers on the beans. What is the sum of all possible values of X?

12. The gingerbread man from Shrek is baking himself a new leg to replace the one that Lord Farquaad broke. He decided to use a template (with the shaded part being his leg). The shaded part's area is equal to half the area of the 5 unit squares. What is the perimeter of his leg?



13. The three pigs, after having their house blown down by the Big Bad Wolf, decide to build a house together in the shape of a polygon $PQRSTUV$. If QR and UV are parallel, the length of PQ equal to VP , and angle PQR equal to 143° , what is the angle of VPQ ?

14. A magical flying fish's path through the water can be modeled $y = x^3 - 5x^2 + 8x - 4$. The moments that the fish goes from the water to the air or vice versa (the three roots of the equation) form the sides of a triangle. The area of the triangle, in simplest form, can be expressed as $\frac{\sqrt{m}}{n}$. What is the sum of the digits of the quantity $(m + n)$?

15. Fibonacci numbers are defined by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_5} + \dots + \frac{F_{20}}{F_{10}}?$$